30th JBMO Buzău România 2026

English

17 June 2026

Problem 1. Let a, b, c be positive real numbers such that

$$a^2 + b^2 + c^2 \geqslant 3.$$

Prove that

$$\frac{a^4}{a^2 + 2b + 2c} + \frac{b^4}{b^2 + 2c + 2a} + \frac{c^4}{c^2 + 2a + 2b} \geqslant \frac{3}{5}.$$

Problem 2. Determine all pairs (a, b) of positive integers such that

$$(a+1) \text{ divides } (b+2) \quad \text{and} \quad b \text{ divides } 2a^2.$$

Problem 3. Let $n \geqslant 3$ be an integer. There are n colored lamps arranged in a circle. Pressing the button of a lamp once changes its color as follows:

$$\text{green} \to \text{red}, \qquad \text{red} \to \text{blue}, \qquad \text{blue} \to \text{green}.$$

Initially, all lamps are colored red. Aladdin makes moves on these lamps. Each move consists of the following three steps:

- he chooses a lamp L, without pressing its button;
- he presses the button of the clockwise neighbor of L once;
- he presses the button of the counterclockwise neighbor of L twice.

For each n, determine the maximum possible number of lamps that are colored green simultaneously, after finitely many moves.

Problem 4. Let ABC be a triangle with $AB \neq AC$ and let I be its incenter. Let P and Q be points inside triangle ABC such that $PB = PC > QC = QB$. Lines BP and CQ meet at X. Suppose that line AI is tangent to the circumcircle of triangle IPQ. Prove that the circumcircles of triangles ABQ, ACP and PQX have a common point.

Time: 4 hours and 30 minutes
Every problem is worth 10 points